quoted. Exposures should, ideally, give a maximum density of 1.0 on the second film of the pack.

Before photometry the film should be carefully scrutinized for any signs (such as irregularities in the spots or unusual shadows in the background) that the experimental arrangements were faulty in any way.

If the photometer errors are comparable with absorption errors the films should be photometered twice, on different occasions, and any measurements which disagree by more than twice the expected standard deviation repeated until consistent results are obtained.

# **Correlation of layers**

Correlation should be achieved by the use of data collected about two axes (Rollett & Sparks, 1960; Hamilton, Rollett & Sparks, 1965). Such data would normally be necessary in any case if all independent reflexions are to be measured. In order to get as many corresponding pairs of reflexions from the two sets as possible and also increase the accuracy of the correlation, the symmetry-related reflexions in the layers should be averaged before correlation (Whitaker, 1965) and the weights already assigned to the intensities (Jeffery & Rose, 1964) adjusted accordingly. If symmetry-related reflexions occur on different films these should be utilized in the correlation.

#### Extinction

Data from more than one crystal should enable extinction to be detected, but for a quantitative estimate some of the single-crystal intensity values should be compared with those measured on a diffractometer using very fine ( $<1\mu$ ) powder, or Chandresekhar's method using polarized X-rays should be applied to the single-crystal specimen.

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# The Crystal Structure of β-In<sub>2</sub>S<sub>3</sub>

# BY G. A. STEIGMANN, H. H. SUTHERLAND AND J. GOODYEAR,

Department of Physics, The University, Hull, England

### (Received 7 April 1965)

A structural analysis of  $\beta$ -In<sub>2</sub>S<sub>3</sub>, using Weissenberg data, has confirmed the cation-deficient spinel structure proposed by Rooymans. The absence of some of the tetrahedrally coordinated cations, found in spinel, leads to appreciable atomic displacements from idealized positions.

### Introduction

On the basis of accurate powder data, Rooymans (1959) proposed that  $\beta$ -In<sub>2</sub>S<sub>3</sub> has a spinel-type structure with ordered vacant tetrahedrally coordinated cation sites. Rooymans found a body-centred tetragonal supercell, containing 16 In<sub>2</sub>S<sub>3</sub> molecules, which was related to the spinel-type cell by  $a=d_{(110)}/2=7.62$  Å and  $c=3d_{(001)}=32.32$  Å,  $d_{(110)}$  and  $d_{(001)}$  being planar spacings of the spinel lattice. The space group suggested by Rooymans was  $I4_{1}22$  with the four cation vacancies per cell located at the equivalent positions of the  $4_{1}$  screw axes.

A more detailed investigation of the space group was undertaken by King (1962) using a single-crystal technique. Systematically absent reflexions on Weissenberg photographs, taken about the [110] axis, suggested the more highly symmetrical space group  $I4_1/amd$ , which is indeed the space group of the structure proposed by Rooymans. King also calculated the unit-cell parameters from the Weissenberg data and found a=7.61, c=32.24 Å.

Goodyear & Steigmann (1961) carried out a singlecrystal examination of twinned crystals of  $\beta$ -In<sub>2</sub>S<sub>3</sub> and were able to explain the twinning mechanism in terms of a cation-deficient spinel model. The twin components of the material available for this work were sufficiently large to enable a suitable single crystal to be selected for the present structural analysis. Accurate powder data published by Goodyear & Steigmann were in agreement with those of Rooymans, and their lattice parameters, a=7.623, c=32.36 Å, have been assumed in this paper.

#### **Intensity measurements**

A single crystal of approximately square cross-section, 0.03 mm  $\times$  0.03 mm, and elongated parallel to the *a* axis was chosen for the analysis. Six layer lines were recorded on equi-inclination Weissenberg photographs taken about the *a* axis with Cu K $\alpha$  radiation.

The intensities of nearly 750 independently recorded reflexions were estimated visually from multiple film exposures with a calibrated scale. To compare the strong principal with the weaker superlattice reflexions  $(l \neq 3n)$ , it was necessary to pack six films into the camera. The systematically absent reflexions were hklwith h+k+l=2n+1, hk0 with h(k)=2n+1, hhl with  $l \neq 4n$ . These are the same conditions as found by King, thus confirming the space group  $I4_1/amd$ . About 150 reflexions were too weak to be observed.

The observed intensities were corrected for Lorentz & polarization factors and adjusted for spot shape on the upper layer lines (Phillips, 1954). Absorption corrections were made by assuming the specimen to be cylindrical, of average radius 0.017 mm with  $\mu r = 1.6$ , and applying the correction factors given by Bond (1959).

## The determination of the structure

Atomic scattering factors listed by Wilson (1960) were used in the intensity calculations.

In the idealized structure the atoms are distributed among the equipoints as follows. Indium in tetrahedral coordination at 8(e), indium in octahedral coordination at 8(c) and 16(h), S in three sets at 16(h). Assuming idealized parameters [Table 1, column (i)] and temperature factors of  $0.8 \text{ Å}^2$  for In and  $1.0 \text{ Å}^2$  for S, initial intensity calculations for 0kl reflexions yielded a value of the reliability index,  $R = \Sigma ||F_0| - |F_c||/\Sigma |F_0|$  of 40%. A close examination of the discrepancy between the observed and calculated structure factors suggested that the principal and superlattice reflexions needed to be scaled separately. When this was done the value of R reduced considerably. At this stage, a two-dimensional least-squares refinement of both atomic coordinates and isotropic temperature factors was carried out with a program by Daly, Stephens & Wheatley (1963) on the Elliot 803 computer installed at the University of Hull. After several cycles of refinement, the *R* index was reduced to  $16\cdot8\%$  for the 0kl reflexions. This was followed by a three-dimensional least-squares refinement with structure factors determined from 741 observed intensities, giving a final *R* value of  $15\cdot4\%$ . The refinement was stopped when the changes in the atomic parameters were significantly less than the standard deviations. As a check on the structure, a (100) electron-density projection was made, and this did not reveal any spurious peaks. The final parameters with their standard deviations are listed in column (ii) of Table 1. The observed and calculated structure factors are compared in Table 2, the mean value of  $|F_o|$  for *hkl* and *khl* reflexions being given. Calculated structure factors of unobserved reflexions were all less than the lowest value of  $|F_o|$ , and these data have not been included in Table 2.

#### Discussion of the structure

The single-crystal data have fully confirmed the basic structure proposed by Rooymans. Since the sequence of atomic layers parallel to the (013) planes of closepacked sulphur atoms has been described previously by Goodyear & Steigmann, it is not proposed to deal with such general features of the structure in this paper, but rather with atomic displacements from idealized spinel positions as revealed by the present analysis. These can be described readily with reference to layers of atoms parallel to the (001) face of the unit cell, as vacant tetrahedral sites are most densely populated in this plane. Fig. 1 shows the directions of the shifts of sulphur and octahedrally coordinated In1, In2, atoms in three successive layers in the unit cell, and of tetrahedrally coordinated In3 atoms situated between the layers.

The In1 atoms are centres of symmetry and, as might be expected, are situated at the ideal positions in the spinel structure. In diagram (a) of Fig. 1 it will be seen that the In1 atoms are linked to S1 atoms, and that the tetrahedral sites on either side of the layer are filled. The displacements of the S1 atoms are thus comparatively small and are mainly due to the large shifts of In3 atoms away from the layer. In the next layer [dia-

Table 1. Atomic parameters and temperature factors

Origin at centre (2/m)

					Oligin a					
	(i) Preliminary				(ii) Final					
Atom	Equipoint	$\overline{x/a}$	y/b	z/c	$\overline{x/a}$	y/b	<i>z/c</i>	$B(Å^2)$		
In1	8(c)	0	0	0	0	0	0	$1.00 \pm 0.06$		
In2	16( <i>h</i> )	0	0	18	0	$-0.0181 \pm 0.0004$	$0.3324 \pm 0.0001$	$1.14 \pm 0.05$		
In3	8(e)	0	4	5 4	0	4	$0.2046 \pm 0.0001$	$1.10 \pm 0.06$		
S1	16( <i>h</i> )	0	0	4	0	$-0.0034 \pm 0.0016$	$0.2506 \pm 0.0003$	1·83 ± 0·21		
S2	16( <i>h</i> )	0	·0	$\frac{1}{12}$	0	$0.0086 \pm 0.0017$	$0.0785 \pm 0.0003$	$1.83 \pm 0.20$		
S3	16( <i>h</i> )	0	0	$\frac{15}{12}$	0	$0.0237 \pm 0.0017$	$0.4131 \pm 0.0004$	1·79±0·19		

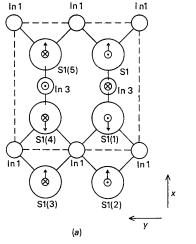
Table 2. Observed and calculated structure factors

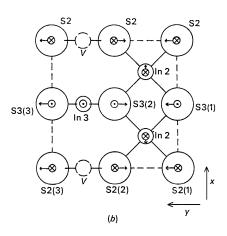
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gram (b)] In2 atoms are linked both to S2 and S3, and the tetrahedral sites above the layer are vacant, each S2 atom being associated with a vacant site. The absence of the tetrahedral bond causes a shortening of the octahedral bonds between S2 and the In2 atoms in the layer and the In1 atom in the first layer. These contractions are accomplished mainly by the large shifts of S2 towards In1, and In2 towards S2 atoms. An S3 atom, being linked octahedrally with three In2 atoms, two in the layer and one in the layer above [diagram (c)], is displaced outwards almost parallel to the tetrahedral bond by the shifts of the In2 atoms. The movements of the S3 atoms, in turn, displace the In3 atom towards the layer. Finally, the same pattern of movement is seen in the next layer [diagram (c)] the displacements now being caused by vacant tetrahedral sites lying below the layer.

The bond lengths of In1-S, In2-S octahedra and In3-S, V-S tetrahedra are given in Table 3. Although the In2-S octahedron is considerably distorted, the average S-S distance is of the same order of magnitude, 3.70 Å compared with 3.72 Å, as that of the In1-S





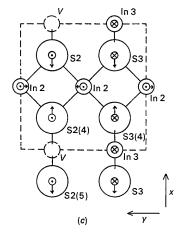


Fig.1. Atomic displacements from idealized position. Shifts parallel to Z are represented by  $\odot$  in direction of Z increasing and  $\otimes$  in direction of Z decreasing. The magnitudes of the shifts (in Å) are:

	Parallel to $X$ (or $Y$ )	Parallel to Z
Inl	0	0
In2	0.14	0.03
In3	0	0.12
<b>S</b> 1	0.03	0.02
S2	0.06	0.16
S3	0.18	0.12

(a) Layer of atoms in unit cell at height z=0 with tetrahedrally coordinated In 3 atoms at z=c/24 and z=-c/24. (b) Layer of atoms in unit cell at height z=c/12 with tetrahedrally coordinated In3 atom z=c/24 and vacant tetrahedral sites, V, at z=c/8. (c) Layer of atoms in unit cell at height z=c/6 with tetrahedrally coordinated In3 atoms at z=5c/24 and vacant tetrahedral sites, V, at tetrahedral sites, V, at z=c/8.

octahedron, and both these values are significantly less than 3.81 Å, the ideal S–S distance. On the other hand, all the S–S distances in both types of tetrahedron are greater than 3.81 Å, the average values being 4.01 Å for an In3–S tetrahedron and 4.04 Å for a vacant tetrahedron.

Standard deviation in each bond length =0.01 Å

In1-S Octahedron		Å
	In-S1(1), $In-S1(2)$	2.68
	In-S2(2)	2.54
	S1(1) - S1(2)	3.76
	S1(1)-S1(4)	3.81
	S1(1)-S2(2), S1(2)-S2(2)	3.64
	S1(3)-S2(2), S1(4)-S2(2)	3.74
In2-S Octahedron		
	In-S2(1), In-S2(2)	2.56
	In-S3(1), In-S3(2)	2.68
	In-S3(4)	2.63
	In-S1(1)	2.65
	S1(1)-S2(1), S1(1)-S2(2)	3.64
	S1(1)-S3(1), S1(1)-S3(2)	3.81
	S2(1)-S3(1), S2(2)-S3(2)	3.82
	S2(1)-S2(2)	3.68
	S3(1)-S3(2)	3.45
	S2(1)-S3(4), S2(2)-S3(4)	3.90
	S3(1)-S3(4), S3(2)-S3(4)	3.47

Table 3 (cont.)

In3-S Tetrahedron		
	In-S1(4), In-S1(5)	2.44
	In-S3(2), $In-S3(3)$	2.48
	S1(4)-S1(5)	3.86
	S1(4)-S3(2), S1(4)-S3(3)	4.01
	S1(5)-S3(2), S1(5)-S3(3)	4.01
	S3(2)-S3(3)	4.17
Vacant tetrahedron		
	S2(1)-S2(3), S2(4)-S2(5)	3.94
	S2(2)-S2(4), S2(2)-S2(5)	4.10
	$S_2(3)-S_2(4), S_2(3)-S_2(5)$	4.10
See F	ig.1 for location of atoms.	

We wish to thank Dr P. J. Wheatley, of Monsanto Research S.A., for providing computer programs which have been used in this work.

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# **Oxygen Coordinates of Compounds with Garnet Structure\***

# BY FERDINAND EULER AND JANE A. BRUCE

Air Force Cambridge Research Laboratories, Office of Aerospace Research, Bedford, Massachusetts, U.S.A.

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By full-matrix least-squares refinement, the O parameters of one natural and twelve synthetic garnets  $M_5R_3O_{12}$  were determined, with M and R as follows:

M:	Al	Ga	Fe
R:	Lu Yb Y Gd	Lu Yb Y	Lu Yb Y Dy Sm.

The refinement was based on those Bragg reflections to which only the oxygen ions contribute. The integrated X-ray intensities were counted after elimination of Renninger effects. The resulting O parameters are interpreted by polar coordinates which indicate size, shape and orientation of the oxygen octahedra and are closely associated with the effects of shortened shared edges on the shape of the other polyhedra. The cell edges *a* are studied as functions of the rare-earth atomic numbers. The crystal field effects on *a*, shown by Espinosa on iron garnets with non-spherical ions R, appear also in the other two series. For spherical R, the increments  $\Delta a$  associated with changes in M and R are independent of each other. Such additivity of *a* is destroyed in Al<sub>5</sub>Gd<sub>3</sub>O<sub>12</sub> by the resistance of the large rare-earth ions to the small lattice. These effects are demonstrated also on selected interionic distances and angles.

## Introduction

As Menzer (1928) has found, the garnet structure belongs to the space group  $Ia3d(O_h^{10})$  with cations in 16(a), 24(c) and 24(d) positions,\* defined by fixed fractions of the cubic cell edge *a*. The 96(h) positions of the anions (oxygen in most garnets) depend on three variable structure parameters *x*, *y*, *z* which, in general, differ with composition.

<sup>\*</sup> Portions of this work were presented at the Pittsburgh Diffraction Conference, Pittsburgh, Pennsylvania, on November 7, 1962.

<sup>\*</sup> Nomenclature of International Tables for X-Ray Crystallography (1952).